December 4, 1991 WOCOMAL VARSITY MEET

ROUND 1 : Arithmetic - percent, interest, discount, fractions, and decimals

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. How much interest is earned in one year on \$3000 at a rate of $5\frac{1}{2}\%$ per year ?

2. What is the product of $0.4\overline{9}$ and $0.\overline{18}$ written as a fraction in lowest terms ?

3. If the length of a rectangle is increased by 50% and the width of the rectangle is increased by 75%, what is the % increase in the area of the rectangle ?

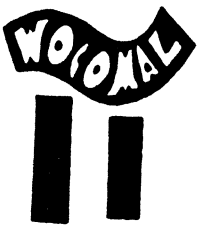
ANSWERS

(1 pt) 1. \$ _____

(2 pts) 2. _____

(3 pts) 3. _____ %

St. John's, Shepherd Hill



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ROUND II: Set theory and logic

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1 In a poll preceding an election, 240 voters liked Bush, 80 liked Quale, and 30 liked both. If every voter who liked either candidate voted for the Bush Quale ticket, how many votes did they get from this group?

2 Each of a group of 50 girls is blond or brunette and is either blue or brown eyed If 14 are blue-eyed blonds, 31 are brunettes, and 18 are brown-eyed, then how many are brown-eyed brunettes?

3 There are eight tennis players entered in a tournament. A player is eliminated after losing two matches. How many matches does the overall winner play?

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

Bromfield, Doherty, Worcester Academy

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ROUND III: Algebra 1 - open

ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST EXACT FORM

1. If $\begin{matrix} a \\ / \quad \backslash \\ b & c \end{matrix}$ is defined to be $ab + ac - bc$, then for what value of x will $\begin{matrix} 3 \\ / \quad \backslash \\ \frac{1}{2} & x \end{matrix}$ be equal to zero?

2. A rectangular lawn is 60 meters by 80 meters. Part of the lawn is torn up to install a rectangular pool, leaving a strip of lawn of uniform width around the pool. The area used for the pool is $\frac{1}{6}$ of the old lawn area. How wide is the strip of lawn?

3. Given $ax + by = 1$, find the equation of the straight line which has twice the x-intercept and twice the slope. Answer in the form $Px + Qy = 1$.

ANSWERS

(1 pt) 1. $x =$ _____

(2 pts) 2. _____ meters

(3 pts) 3. _____

Tantasqua, Westborough, Worcester Academy

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ROUND IV: Sequences and series

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Evaluate $\sum_{n=2}^{7}(3n-2)$

2. If five geometric means are inserted between 8 and 5832, find the fifth term in the geometric sequence that starts with 8.

3. In a certain arithmetic sequence with first term 3, the ratio of the fourth term to the ninth term is equal to the ratio of the seventh term to the sixteenth term. Find the tenth term.

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

Bartlett, Bromfield, Hudson

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ROUND V: Matrix and determinant operations

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. If $A = \begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$ and A^T is its transpose, find the product $(-A)(A^T)$.

2. $B = \begin{bmatrix} 3 & 1 \\ 6 & 4 \end{bmatrix}$. I is the 2×2 multiplicative identity matrix and x is a scalar. Solve the determinant equation

$$|B - xI| = 0$$

for x.

3. Suppose A^{-1} is the symbol for the multiplicative inverse of matrix A. Also suppose that $AX = B$ and $YA = B$. Express XY in terms of A^{-1} and B.

ANSWERS

(1 pt) 1. $\begin{bmatrix} \quad \end{bmatrix}$

(2 pts) 2. _____

(3 pts) 3. _____

Quaboag, South, Worcester Academy

TEAM ROUND: Topics of previous rounds and open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM
AND ON THE SEPARATE TEAM ANSWER SHEET 2 points each

1 Find the result as a reduced fraction if $\frac{1}{2}\%$ of $\frac{1}{3}$ is divided by $\frac{1}{4}$

2. Set X has 63 proper subsets. Set Y has 15 proper subsets. $X \cup Y$ has 127 proper subsets. How many proper subsets has $X \cap Y$?

3 Simplify $(1+x-x^3)(1-x+x^2) - (1+x+x^2+x^3+x^4)(1-x)$.

4 If $\sum_{k=1}^{n} g(k) = \frac{n}{3n-2}$, find $g(5)$.

5. If for these determinants

$$\begin{vmatrix} a & d & g \\ b & e & h \\ c & f & i \end{vmatrix} = k \begin{vmatrix} pd & a & g+ma \\ pe & b & h+mb \\ pf & c & i+mc \end{vmatrix},$$

find the value of k in terms of p and m.

6. Evaluate
$(33.1068)(433.186) + (2602.22)(3.31068) + (.0331068)(306592)$.

7 There are ten children in a room. The ratio of boys to girls increased when another boy and another girl entered the room. What is the largest number of boys that could have been in the room originally?

8. $A_1, A_2, A_3, \ldots$ is an arithmetic sequence of 2×2 matrices with respect to matrix addition. If $A_4 = \begin{bmatrix} 1 & 4 \\ 9 & -5 \end{bmatrix}$ and $A_7 = \begin{bmatrix} -5 & 1 \\ 3 & 8 \end{bmatrix}$, find A_1.

9 Which is the largest of these numbers? $1^{\frac{1}{2}}$, $2^{\frac{1}{3}}$, $3^{\frac{1}{4}}$, $4^{\frac{1}{5}}$, and $5^{\frac{1}{6}}$

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ROUND I arith

1. $165
2. $\frac{1}{11}$
3. 162.5 %

ROUND II sets, logic

1. 290
2. 13
3. 5 or 6 (need both)

ROUND III alg 1

1. $-\frac{3}{5}$ or -0.6
2. 20
3. $\frac{a}{2}x + \frac{b}{4}y = 1$ ($\frac{1}{2}ax$ etc. ok but not $\ldots = 4$)

ROUND IV seq. series

1. 69
2. 648
3. 39

ROUND V matrix, det.

1. $\begin{bmatrix} -20 & -14 \\ -14 & -10 \end{bmatrix}$
2. 1 or 6 (need both)
3. $A^{-1}B^2A^{-1}$ or $A^{-1}BBA^{-1}$

TEAM ROUND

1. $\frac{1}{150}$
2. 7
3. x^4
4. $-\frac{1}{65}$
5. $\frac{-1}{p}$
6. 33 106.8
7. 4
8. $\begin{bmatrix} 7 & 7 \\ 15 & -18 \end{bmatrix}$
9. $4\frac{1}{5}$